

objectives

- 1) Write an exponential equation as an equivalent logarithmic equation.
- 2) Write a logarithmic equation as an equivalent exponential equation.
- 3) Solve logarithmic equations which can be written as exponential equations with a common base
- 4) Evaluate log expressions
- 5) Graph logarithmic functions.
- 6) Identify properties of logs (any valid base b)

$$\log_b(1) = 0$$

$$\log_b(b) = 1$$

$$\text{domain of } f(x) = \log_b(x) \quad x > 0 \text{ or } (0, \infty)$$

$$\text{range of } f(x) = \log_b(x) \quad \text{all reals or } (-\infty, \infty)$$

- 7) Use the fact that the log and exponential functions with the same base are inverses

$$f(f^{-1}(x)) = x$$

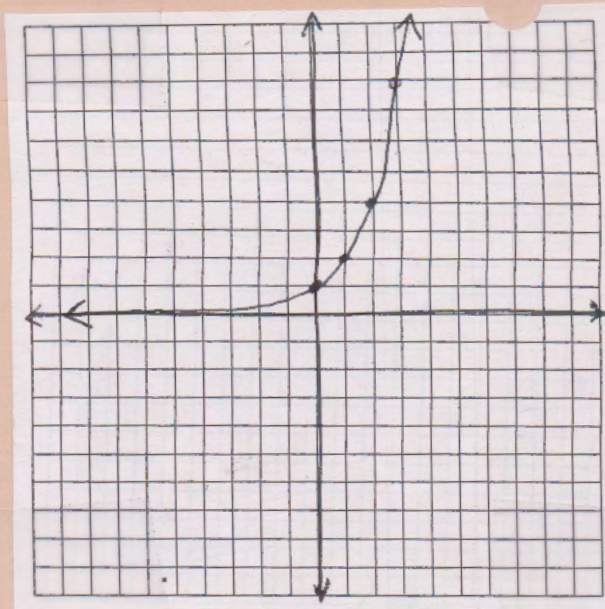
$$\log_b b^k = k$$

$$f^{-1}(f(x)) = x$$

$$b^{\log_b k} = k$$

Recall: Exponential function
 $f(x) = 2^x$

x	$f(x)$
-3	$\frac{1}{8}$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8



Passes V.L.T. (it's a function)
 Passes H.L.T. (it's one-to-one)
 So it's invertible!

x	$f^{-1}(x)$
$\frac{1}{8}$	-3
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2
8	3

But we don't have any algebra to describe $f^{-1}(x)$!

This function transcends (surpasses) algebra, so it's called a transcendental function.

But what is it?

We know that if $f(x) = 2^x$ then

$$\begin{aligned} f^{-1}\left(\frac{1}{8}\right) &= -3 \\ f^{-1}\left(\frac{1}{4}\right) &= -2 \\ f^{-1}\left(\frac{1}{2}\right) &= -1 \\ f^{-1}(1) &= 0 \\ f^{-1}(2) &= 1 \\ f^{-1}(4) &= 2 \\ f^{-1}(8) &= 3 \end{aligned}$$

The notation mathematicians created for an $f^{-1}(x)$ of this type is called a logarithm.

$$f^{-1}(x) = \log_2(x) \text{ or just } \log_2 x$$

Pronounce: "log-base-2-of-x"

Write the 2 below the log, as a subscript.
The base is crucial.

① If $f(x) = 3^x$, what is $f^{-1}(x)$?

$$\boxed{f^{-1}(x) = \log_3 x}$$

② If $f(x) = \log_4 x$, what is $f^{-1}(x)$?

$$\boxed{f^{-1}(x) = 4^x}$$

More terminology: $y = \log_2(x)$

x is the input, also called argument

2 is the base

y is the logarithm or the value of the logarithm

So if (a, c) is a pair on the graph of $f(x) = b^x$ $c = b^a$, then
this means (c, a) is a pair on the graph of $f^{-1}(x) = \log_b x$ $a = \log_b c$

$c = b^a$
exponential eqn

is equivalent to

$a = \log_b c$
logarithmic equation

Write each exponential equation as an equivalent logarithmic equation.

③ $2^3 = 8$

$\log_2 8 = 3$

④ $9^3 = 729$

$\log_9 729 = 3$

⑤ $6^{-2} = \frac{1}{36}$

$\log_6 \left(\frac{1}{36}\right) = -2$

⑥ $\sqrt[3]{5} = 5^{\frac{1}{3}}$

$\log_5 \sqrt[3]{5} = \frac{1}{3}$

⑦ $\pi^4 = x$

$\log_\pi x = 4$

Common mistake

Since 2 and 3 are both on the LHS of the exponential, it's tempting to put both 2 and 3 on the LHS of the logarithmic, but that's wrong.

Write each logarithmic equation as an exponential equation.

⑧ $\log_5 25 = 2$

$5^2 = 25$

⑨ $\log_6 \frac{1}{6} = -1$

$6^{-1} = \frac{1}{6}$

⑩ $\log_2 \sqrt{2} = \frac{1}{2}$

$2^{\frac{1}{2}} = \sqrt{2}$

⑪ $\log_7 x = 5$

$7^5 = x$

⑫ $\log_{13} y = 4$

$13^4 = y$

If all the parts are numbers, you should get a true statement.

Find the value of each logarithmic expression.

⑬ $\log_4 16$

step 1: Set expression equal to a variable you choose

$\log_4 16 = Q$

step 2: Write equivalent exponential equation

$4^Q = 16$

step 3: Solve the exponential equation by writing both sides with a common base, then set exponents =.

$$4^Q = 4^2$$

$$Q = 2$$

$$\boxed{\log_4 16 = 2}$$

* Work toward doing these steps in your head*
 $\log_4 16$ means 4 raised to what equals 16?

$$(14) \log_{10} \left(\frac{1}{10} \right) = Q$$

$$10^Q = \frac{1}{10}$$

$$10^Q = 10^{-1}$$

$$Q = \boxed{-1}$$

or 10 raised to the what equals $\frac{1}{10}$?

$$(15) \log_9 3 = Q$$

$$9^Q = 3$$

$$(3^2)^Q = 3^1$$

$$2Q = 1$$

$$Q = \boxed{\frac{1}{2}}$$

$$\text{or } 9^{1/2} = 3$$

$$(16) \log_3 9 = Q$$

$$3^Q = 9$$

$$3^Q = 3^2$$

$$Q = \boxed{2}$$

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Solve each equation for the unknown variable.

(17) $\log_4 \frac{1}{4} = x$

Step 1: write equivalent exponential equation.

$$4^x = \frac{1}{4}$$

$$4^x = 4^{-1}$$

$$\boxed{x = -1}$$

(18) $\log_5 x = 3$

Step 1: write equivalent exponential equation

$$5^3 = x$$

$$\boxed{x = 125}$$

(19) $\log_x 25 = 2$

$$x^2 = 25$$

$$x = \pm 5$$

However... a base can only be a positive number

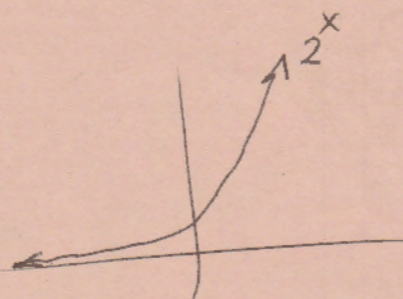
So $x = -5$ is extraneous

$$\boxed{x = 5}$$

(20) $\log_2 (-2) = x$

$$2^x = (-2)$$

has $\boxed{\text{no solution}}$



2 raised to any number is always positive

(21) $\log_3 1 = x$

$$3^x = 1$$

$$3^x = 3^0$$

$$\boxed{x = 0}$$

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(22) $\log_b 1 = x$ solve for x

$$b^x = 1$$

$$b^x = b^0$$

for any valid base $b > 0, b \neq 1$.

$$\boxed{x = 0}$$

(23) $\log_x \frac{1}{7} = \frac{1}{2}$

$$x^{\frac{1}{2}} = \frac{1}{7}$$

$$(x^{\frac{1}{2}})^2 = \left(\frac{1}{7}\right)^2$$

$$\boxed{x = \frac{1}{49}}$$

(24) $\log_4 2^4 = x$

$$4^x = 2^4$$

$$(2^2)^x = 2^4$$

$$2^{2x} = 2^4$$

$$2x = 4$$

$$\boxed{x = 2}$$

(25) $\log_{\frac{3}{4}} x = 3$

$$\left(\frac{3}{4}\right)^3 = x$$

$$\boxed{x = \frac{27}{64}}$$

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(26) If $f(x) = 4^x$ find $f^{-1}(x)$.

$$\boxed{f^{-1}(x) = \log_4 x}$$

(27) If $g(x) = \log_3 x$, find $g^{-1}(x)$

$$\boxed{g^{-1}(x) = 3^x}$$

(28) If $f(x) = 5^x$, write $f(f^{-1}(x))$ unsimplified and simplified.

$$f(x) = 5^x$$

$$f^{-1}(x) = \log_5 x$$

$$\boxed{\begin{array}{l} f(f^{-1}(x)) = 5^{\log_5 x} \\ f(f^{-1}(x)) = x \\ \text{so } 5^{\log_5 x} = x \end{array}}$$

unsimplified

for all inverse functions

(29) If $f(x) = 5^x$, write $f^{-1}(f(x))$.

$$f^{-1}(f(x)) = \boxed{\log_5(5^x) = x}$$

Math 70. M-G 9.5 Log Functions, Day 2

Recall:

If $f(x) = b^x$

$f^{-1}(x) = \log_b(x) = \log_b x$

logs + exponentials with the same base are inverses of each other

or if $f(x) = \log_b x$

$f^{-1}(x) = b^x$

Solve the equations.

① $\log_2 1 = x$

② $\log_{1/3} 1 = x$

③ $\log_{10} 1 = x$

④ $\log_{\pi} 1 = x$

$x=0$
for
all

Property #1

$\log_b 1 = 0$

for any valid base b .
($b > 0, b \neq 1$)

Solve the equations:

⑤ $\log_2 2^5 = x$

$x=5$

⑥ $\log_{1/3} (1/3)^{-4} = x$

$x=-4$

⑦ $\log_{10} 10^2 = x$

$x=2$

⑧ $\log_{\pi} \pi^{-1} = x$

$x=-1$

Property #2

$\log_b b^x = x$

for any valid base b .
($b > 0, b \neq 1$)

Inverse function property

⑨ If $f(x) = 2^x$ and $f^{-1}(x) = \log_2 x$, write the expression

for $(f^{-1} \circ f)(x)$.

$= f^{-1}(f(x))$

$= f^{-1}(2^x)$

$= \log_2 2^x = x$.

$(f^{-1} \circ f)(x) = x$.

That's what inverse functions are supposed to do!

(10) If $f(x) = 2^x$ and $f^{-1}(x) = \log_2 x$, write the expression for $(f \circ f^{-1})(x)$.

$$\begin{aligned} & f(f^{-1}(x)) \\ &= f(\log_2 x) \\ &= 2^{\log_2 x} = x \end{aligned}$$

But if these are really inverses, this is x !

Property #3:

$$b^{\log_b x} = x \quad \text{for any valid base } b$$

Inverse function property

Simplify:

(11) $\log_3 3^2 = 2$

(12) $\log_7 7^{-1} = -1$

(13) $5^{\log_5 3} = 3$

(14) $2^{\log_2 6} = 6$

Sketch graphs of these logarithmic functions.

* CAUTION * #1 Just plotting points from the usual table of x-values is not enough.

* CAUTION * #2 Just graphing what you see on your GC is not enough.

* CAUTION * #3 Your GC only has $\log_{10}(x)$ and $\log_e(x)$ anyway.

(15) $y = \log_2 x$

step 1: Find the equivalent exponential function and make a table of values for it.
Clearly label it exponential.

x	2^x
-4	$\frac{1}{16}$
-3	$\frac{1}{8}$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8
4	16

x	$\log_2 x$
$\frac{1}{16}$	-4
$\frac{1}{8}$	-3
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2
8	3
16	4

step 2: Swap all the ordered pairs (a,b) on exponential graph to get ordered pairs (b,a) on the logarithmic graph. Clearly label it logarithmic.
Cross out the exponential table if necessary

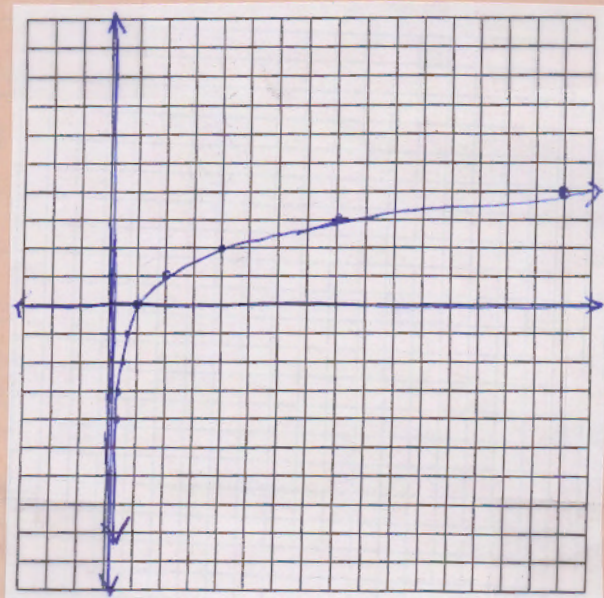
step 3: Plot the swapped points.

step 4: Check that 3 areas are graphed and clear

- 1) logarithmic growth
- 2) x-intercept
- 3) vertical asymptote.

domain: $x > 0$ or $(0, \infty)$

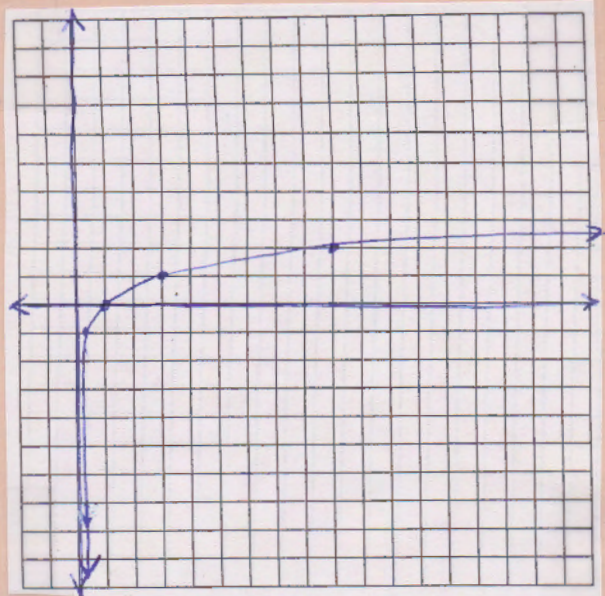
range: all reals or $(-\infty, \infty)$



⑩ $y = \log_3 x$

x	3^x
-4	$\frac{1}{81}$
-3	$\frac{1}{27}$
-2	$\frac{1}{9}$
-1	$\frac{1}{3}$
0	1
1	3
2	9
3	27
4	81

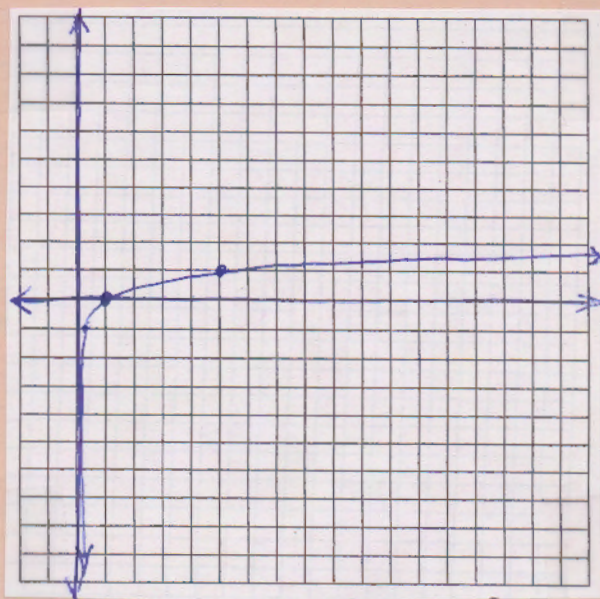
x	$\log_3 x$
$\frac{1}{81}$	-4
$\frac{1}{27}$	-3
$\frac{1}{9}$	-2
$\frac{1}{3}$	-1
1	0
3	1
9	2
27	3
81	4



⑪ $y = \log_5 x$

x	5^x
-3	$\frac{1}{125}$
-2	$\frac{1}{25}$
-1	$\frac{1}{5}$
0	1
1	5
2	25
3	125

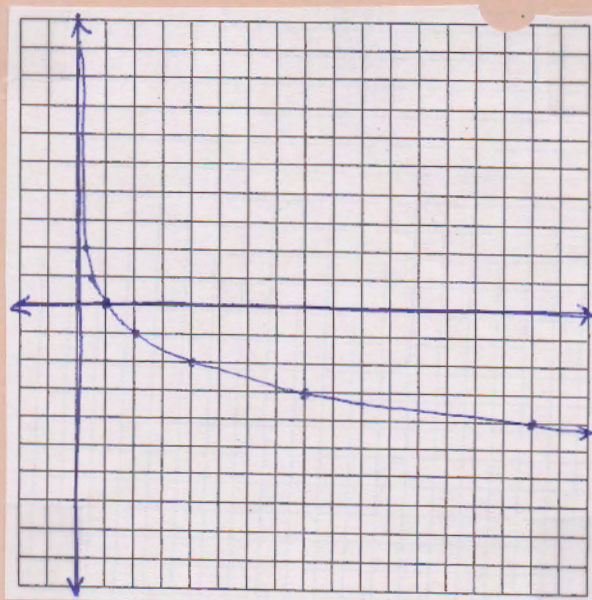
x	$\log_5 x$
$\frac{1}{125}$	-3
$\frac{1}{25}$	-2
$\frac{1}{5}$	-1
1	0
5	1
25	2
125	3



⑫ $y = \log_{1/2} x$

x	$(\frac{1}{2})^x$
-4	$(\frac{1}{2})^{-4} = 2^4 = 16$
-3	8
-2	4
-1	2
0	1
1	$\frac{1}{2}$
2	$\frac{1}{4}$
3	$\frac{1}{8}$
4	$\frac{1}{16}$

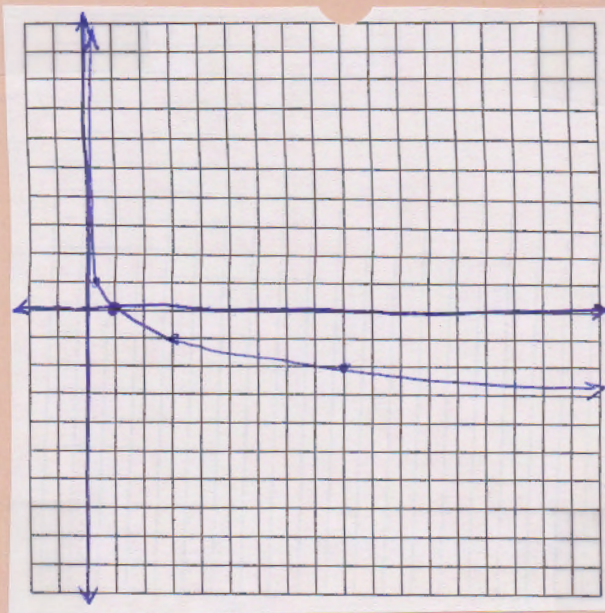
x	$\log_{1/2} x$
16	-4
8	-3
4	-2
2	-1
1	0
$\frac{1}{2}$	1
$\frac{1}{4}$	2
$\frac{1}{8}$	3
$\frac{1}{16}$	4



(19) $y = \log_{1/3} x$

x	$(1/3)^x$
-4	$(1/3)^{-4} = (3)^4 = 81$
-3	$(1/3)^{-3} = 3^3 = 27$
-2	9
-1	3
0	1
1	$1/3$
2	$1/9$
3	$1/27$
4	$1/81$

x	$\log_{1/3} x$
81	-4
27	-3
9	-2
3	-1
1	0
$1/3$	1
$1/9$	2
$1/27$	3
$1/81$	4



(20) $y = -\log_2 x$

step 1: Make table for 2^x

step 2: Swap to get table for $\log_2 x$

step 3: Keep all x-coords of $\log_2 x$ table, but make all y-coords opposite

x	2^x
-4	$1/16$
-3	$1/8$
-2	$1/4$
-1	$1/2$
0	1
1	2
2	4
3	8
4	16

x	$\log_2 x$
$1/16$	-4
$1/8$	-3
$1/4$	-2
$1/2$	-1
1	0
2	1
4	2
8	3
16	4

x	$-\log_2 x$
$1/16$	+4
$1/8$	+3
$1/4$	+2
$1/2$	+1
1	0
2	-1
4	-2
8	-3
16	-4

